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Efficiency of Runge-Kutta methods in solving Kepler problem

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KEYWORDS

Phase diagrams

Exact solutions

ABSTRACT

The aim of this research is to study the efficiency of symplectic and non-symplectic Runge-Kutta methods in solving Kepler problem. The numerical behavior of the Runge-Kutta (RK) methods that are symmetric such as the implicit midpoint rule (IMR), implicit trapezoidal rule (ITR), 2-stage and 2-stage Gauss (G2) method are compared with the non-symmetric Runge-Kutta methods such as the explicit and implicit Euler (EE and IE), explicit midpoint rule (EIMR), explicit trapezoidal rules (EITR), explicit 4-stage Runge-Kutta (RK4) method and 2-stage Radau IIA method (R2A). Kepler problem is one type of nonlinear Hamiltonian problem that describes the motion in a plane of a material point that is attracted towards the origin with a force inversely proportional to the distance squared. The exact solutions phase diagram produces a unit circle. The non-symplectic methods only reproduce a unit circle at certain time intervals while the symplectic methods do produce a unit circle at any time intervals. Some phase diagram show spiral in or spiral out patterns which means the solutions are running away from the unit circle. This also means that the absolute error will be increasing in long time integration. The numerical experiments for the Kepler problem are given for many time intervals and the results show that the most efficient method is G2 of order-4 and surprisingly RK4 seems to be efficient too although it is not a symplectic nor a symmetric method. The numerical results on Kepler problem concluded that, the higher the order of the method, the most efficient the method can be in solving Kepler problem despite whether they are explicit or implicit or symmetric and symplectic.

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